The General Theory of Relativity and World Around Us

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So we are back again to Einstein's work on relativity. After reading so many things of the special theory of relativity one might justifiably rest content that it was enough. Let us digest the facts that at higher speeds lengths of rigid bodies will shorten, time will run slowly, mass will absorb energy and increase, velocities will add up but too hesitantly to exceed the velocity of light, so on and so forth. Let us also concede that space and time measurements are always relative; that there is no absolute space in absolute rest, nor any absolute and universal flow of time; that space and time are not separate from and independent of each other but intertwined together into a four-dimensional space-time continuum.

But the problem is that Einstein was not satisfied even with all that. He found some limitations in his first work (STR) on the kinematics of bodies, for he was bothered by the question why the laws of physics should be valid only in the inertial frames of reference, and not on any kind of co-ordinate systems in general, like rotational or accelerated frames. So he engaged himself in research to go still further and finally, in 1916, published his paper on the general theory of relativity (GTR).[1] We shall therefore follow him in the further excursions of the subject of relativity.

Equivalence of the Two Masses

Like the famous heroes in history, the theory of relativity also had been a rich source of many a mythology – centring round the question of its clarity.

Here are two samples:

A student came to Eddington and reported to him with an exclamation that the theory of relativity, he heard, was so difficult that only three persons in the world could understand it. Eddington wondered and asked the student: "Who is the third man, my boy?"

Richard Feynman was a bit more liberal. He conceded that twelve men understood the relativity theory.

In the centenary year of the theory perhaps they would allow more people to understand it. But a halo of abstruseness is always associated with this theory. Even Einstein expressed his disgust on several occasions when he found many qualified people talking nonsense on the subject of relativity, specially on his general theory. Our first task would therefore be to dispel, as much as possible, this obscurity and try to get at the essence of general relativity by walking with Einstein in his gradual venture into the subject. Then we may discover that we can have a grasp of the physical import of GTR, even without its highly abstract mathematics.

There were certain conceptual riddles concerning some facts in physics ever since Newton's time. Since the physicists of the two centuries after Newton could not solve the conceptual problems nor assign any cause for them, they attributed the facts to mere accident. For example, the equality of the inertial mass and the gravitational mass of a body was known to every body but could not be accounted

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for. So it was thought to be a fortuitous matter.

Let us explain.

When a body is impelled by a force to produce an acceleration, it offers a resistance owing to its inertia, which is proportional to its mass. The force is required to overcome the inertia. For the same mass, production of a higher magnitude of acceleration will require a higher force. On the other hand, for a given acceleration, the greater is the mass, the greater the inertia, and hence the greater the force to be applied. In other words, the force then is proportional to mass. Newton's second law of motion made it explicit in a simple mathematical form:

$$F = m.a.$$

Now, if the same body is thrown downward in open air from a height, it will be subjected to the attractive force of the earth's gravity. Earlier, from the time of Aristotle, from insufficient empirical data, men had thought that different bodies fell from the same height to the ground in different times, that is, with different accelerations. The heavier body fell the more quickly than the lighter one. Most of the people, who made at all any observation, did it with a piece of stone and a leaf or a feather. And they got confirmed in their belief. It required the genius of – yes, once again Galileo – to question the generality of this observation by suggesting to drop two pieces of stones from the same height together, one perceptibly heavier than the other, and to consider the time taken by each to fall to the ground. From these studies Galileo formulated his laws of falling bodies.

Newton deduced a simple result from these laws. When two bodies fall freely from the same height (ignoring the resistance of air), the fact that they take the same time to reach the ground, Implies that they fall with the same acceleration. This is quite evident from the following consideration.

From Newton's laws of motion,

$$h = u_1t + ½\, a_1t^2,$$

$$\text{and, } h = u_2t + ½\, a_2t^2,$$

where a_1 and a_2 are (if possible, different) accelerations of the two bodies.

Let the bodies fall from rest, so that $u_1 = u_2 = 0$ in both cases.

$$\text{Then } a_1 = a_2.$$

This implies on generalization that all bodies fall to the ground with the same acceleration, which is independent of their masses.

Newton defined this as the acceleration due to gravity of the earth (g) and suggested a similar formula for a force with which this acceleration is associated:

$$W = m.g.$$

Comparing this relation with the general formula for force and acceleration Newton found that these two quantities for mass are same for the same force and same acceleration in any direction.

Let us clarify.

If a force W produces in the horizontal direction an acceleration g on a body of inertial mass m, and if a gravitational pull equal to W produces the vertically downward acceleration due to gravity g on the same body of gravitational mass m', then by virtue of Newton's laws,

$$W = mg,$$
$$\text{and, } W = m'g.$$
$$\text{Obviously, } m = m'.$$

However, Newton or any body after him till Einstein did not question why this was so, nor could they theoretically account for this fact. They had all taken it for granted, as a whim of Nature.

Einstein rightly understood that this could not be accidental but there should have been some profound clue to the mystery of nature underlying this equality. In fact, it pointed him, as we shall see later, to the equivalence of the gravitational motion with the general kinematics of bodies. So he sought for an explanation.

He reasoned in two ways.

[1] A freely falling body can be approached from two angles. Viewed from the side of the earth, there is a "calling force of the earth", proportional to the 'heaviness' (gravitational mass) of the body, which "responds to the call". Taking the body in isolation from the earth, it is experiencing acceleration and is therefore subject to a mechanical force proportional to the 'inertia' (inertial mass) of the body, which offers resistance to the motion. Since these two pictures represent the same motion of the same body under the same force, two masses should be equal.

[2] It could be conceived in a more technical manner as follows: The acceleration of a freely falling body might be supposed to increase in proportion to its heaviness (gravitational mass, which responds to the gravitational pull of the earth), and to decrease in proportion to its inertia (inertial mass, which resists the motion in the direction of the operating force). Since the acceleration remained constant throughout the fall, the gravitational mass and the inertial mass could not but be equal.[2]

Having shown the reason of the equality of the two masses, Einstein looked into its physical significance: "*The same* quality of a body manifests itself according to circumstances as "inertia" or as "weight" (lit. "heaviness")."[3] If mass is fundamentally same in the case of a mechanical action as well as under the gravitational pull, gravitation cannot be fundamentally different from mechanical actions.

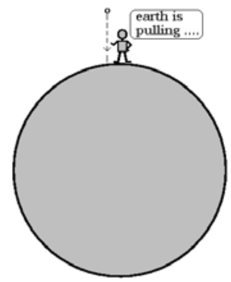

Fig. 1(a)

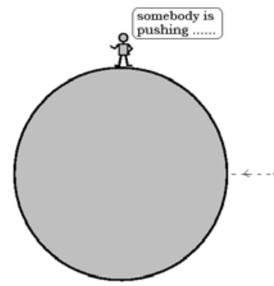

Fig. 1(b)

In other words, in the larger canvas of the universe, Einstein argued, the two accelerated motions are equivalent. The vertical acceleration cannot be distinguished from the horizontal; the two are relative and depends only on the choice of the viewing positions.

To grasp the point, consider the case of the earth and its inhabitants on the equator and at any of the pole. The upward throw of a ball in the polar region will appear sidewise to the equatorial people. And vice versa (see figs. 1a and 1b). But at every point on the earth surface the concerned observer himself will see upward and downward motion of the ball thrown by him. Naturally the same is true in still larger scale of the cosmological space.

Moreover, since the acceleration of all freely falling bodies are same and independent of the nature of the bodies concerned, this constancy of acceleration represents the most important aspect of the property called gravitation. In other words, the motion of a body under gravitation must be viewed as equivalent to its motion with a constant acceleration.

Action at a Distance?

There was, however, another problem to be solved before he could conclusively dissolve the difference between them.

There was a difficulty to regard gravitation as a mechanical action.

Newton's theory of gravitation presupposed two things: first of all, existence of at least two bodies exerting attractive influences on each other, and secondly, *action at a distance*. For example, according to this theory the sun is exerting a force of attraction on each of the planets, which are approximately 10^8–10^{10} kilometers away, and the action is carried at infinite speed instantaneously from the sun to the planets. The planets are also in their turn exerting forces of attraction on the sun as well as on one another. And the problem of action at a distance applies equally there too.

Already the STR had exposed that it was impossible even for the sun to send its pulling notice to the planets in this way. For, no physical signal, influence, or information could travel faster than the velocity of light in vacuum.

On the other hand, in the field of mechanics, the action of a force takes place locally, and is transferred through physical contact from one body to another. Moreover, it takes finite time for a force to act and to move a body from one point to another.

In these formal aspects, therefore, the mechanical action and the gravitational pull were evidently incompatible.

Fields and Oscillations

Einstein, however, noted that the way to remove the incompatibility was being steered clear for long in a different place – the theoretical development of the electromagnetic field and wave through the works of Örsted, Faraday, Lenz, Maxwell and Hertz. They discovered that the electrical and magnetic effects spread through space as electrical and magnetic fields respectively, in the forms of local disturbances in the field intensity called oscillations, which run at a constant speed equal to that of light in empty space. Fields are material reality and represent the distribution of energy radiated by a magnet or an electrical charge. And this distribution at any point of the field depends directly on the charges or the pole-strengths and inversely on the square of the distance of the point from the energy radiating source. The magnetic or electrical force experienced by another magnetic pole or another electrical charge brought at a point in this field is given as:

$$F \propto q_1.q_2/d^2$$

In this respect these are already known to be quite akin to the mathematical forms of the gravitational influence developed by Newton:

$$F \propto m_1.m_2/d^2$$

But the theory of magnetic or electrical effects does not require any action at a distance. Nor does it presuppose the existence of at least two bodies to exert mutual attraction. Field potential exists at a point irrespective of there being any body there to experience it.

It was then further discovered that the spread of light, heat, etc. in vacuum is similar to the propagation of the electrical impulse following Maxwell's equation. Maxwell therefore boldly concluded from this that light is also a form of electromagnetic radiation. So light can propagate as the movement of oscillations in the field caused by the luminal source without necessarily requiring any body else to experience the impacts of these oscillations.

But every body is not as bold as Maxwell to see the general in the particular. So the physicists after him could not just take the immediate follow-up during the next fifty years till Einstein.

Earlier, the mathematical form of the force law of the electrical and magnetic fields was inspired by the Newton's law of gravitation. Now Einstein's genius immediately saw the required clue for gravitation in the field theory of electromagnetism. If gravitation of a body could also be explained in terms of field effects, both the difficulties of Newtonian theory could be overcome.

Einstein wrote: "In Maxwell's theory there are no material actors. The mathematical equations of this theory express the laws governing the electromagnetic field. They do not, as in Newton's laws, connect two widely separated events; they do not connect the happenings *here* with the conditions *there*. The field *here* and *now* depends on the field in the *immediate neighbourhood* at a time *just past*. The equations allow us to predict what will happen a little farther in space and a little later in time, if we know what happens here and now. They allow us to increase our knowledge of the field by small steps. We can deduce what happens here from that which happened far away by the summation of these very small steps."[4]

Field actions can spread through local disturbances and therefore do not involve any action at a distance. And secondly, in that case gravitational influence may be conceived as existing even when no body is experiencing it. On the other hand if a body enters this gravitational field, it will surely be subject to its influence.

But representation of gravitation in the form of a field picture requires two things: elaboration of the behaviour of a body in such a field, and elucidation of the character of this field.

Accelerated Frame, or, Gravitation?

Einstein had already embarked on the first task when he sought for the significance of the equality of the inertial and gravitational mass. Now he proceeded to get to the logical sequel to this equality by showing equivalence between the behaviours of a body in an accelerated frame of reference and in a gravitational field through some ideal thought experiments as follows.[5]

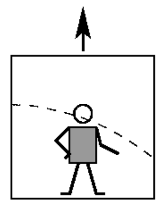

Fig. 2(a)

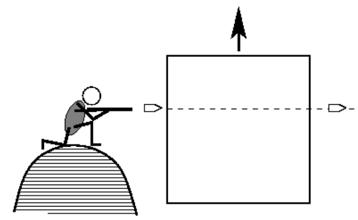

Fig. 2(b)

Experiment-1: Suppose, we are in a faraway remote corner of the cosmos, which is practically free of any gravitational pull of any cosmological object. There, suppose, a lift made of glass is being pulled above by some arrangement with a constant acceleration; and a passenger, born and brought up there is living inside it. When the lift passes by a cliff, suppose an expert shooter shoots a rifle and the powerful bullet pierces the glass wall of the lift from the cliff-side, goes through it and comes out of the lift form the opposite wall. What will the passenger inside the lift see about the course of motion of the bullet? And what will the shooter observe?

The passenger will see the bullet traverse a curved and downward path (see fig. 2a) as in the case of Newtonian gravitation. For the more the bullet advances in a straight line, the higher the floor of the lift rises. The relative motion of the bullet will therefore appear to him to be in a parabolic path.

The shooter will of course see the bullet go in a straight line through the lift. However, because of the upward motion of the lift, he will see that the bullet emerges through a lower point of the opposite wall than the level of the point of the near wall through which it goes in (fig.2b). His belief in Newton's laws of motion will be reinforced.

But if the inside man knows nothing about the outside world, he will naturally try to explain the behaviour of the bullet in terms of gravitation *per se* Newton. And he is not wrong about that.

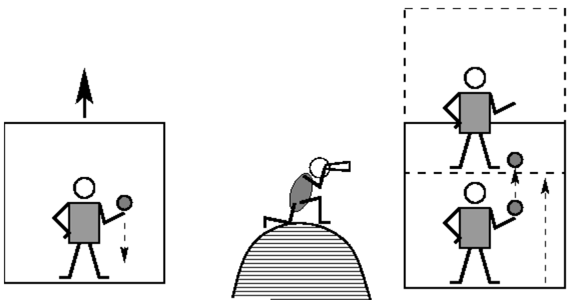

Fig. 3 (a) Fig. 3(b)

Experiment-2: For he may confirm his idea with a simple experiment by softly leaving a ball out of his grip. He will then see the ball gradually and more and more rapidly dropping down to the floor of the lift (fig. 3a). If he jumps high, he will feel falling down on the floor. In this way, he will feel he is glued to an inertial frame under the spell of gravitation. His situation is in many respects similar to ours on the earth surface.

On the other hand, if the outside shooter observes this, he will say: No, both the ball and the lift are rising upwards; but the lift because of its acceleration is going to overtake the ball which is rising with a constant speed, at the speed with which it was rising in the grip of the passenger before it was left out. That is why the floor rises more rapidly and bumps against the ball (fig. 3b). Similarly, the jumping of the passenger will appear to him as the floor overtaking the man.

These two experiments make it amply clear that both the observers are right about what they see and say from their corresponding frames of reference. The same fact, which the insider observes as gravitation in an inertial frame of reference, for he knows nothing about the accelerated motion of the lift, the outsider observes as a displacement in an accelerated frame of reference.

Experiment-3: Now suppose, the same lift is taken back to the neighbourhood of a large star and dropped down in a free fall without the knowledge of the passenger. If he now lets the ball out from his grip, he will see it, as it were, floating within the atmosphere of the lift. If he applies a push on the ball, he will see the ball move with a constant velocity as in an inertial frame of reference. Whatever experiments he conducts in the lift, it will reveal no motion of the lift, like the Ship of Galileo on a calm sea. From all this he may rightly conclude that he is in an ideal inertial Galilean frame, and all the laws of physics are valid in it. (Remember that this is an inertial or Galilean co-ordinate system in a very limited part of the space – "a pocket edition" of the inertial system, so to say.[6] But it serves our main purpose quite sufficiently.)

Now what will an outside observer see? He will explain every thing in terms of gravitation, for that is how he sees things go. Both the lift and the ball are falling down with the same acceleration due to the gravitational pull of the star, as it happens on the surface of the earth. That is why they remain in the same position with respect to each other. And the laws of physics are as valid there as on the surface of the earth.

From all these thought experiments Einstein came to the firm conclusion that the behaviour of a body in a gravitational field attached to an inertial frame of reference is similar to its being in an accelerated frame of reference. What appears as gravitation in one frame of reference will appear as accelerated motion in another. And in the cosmological space, all directions are isotropic and symmetrical; as we have already seen, an accelerated motion in any direction is equivalent to a description of an upward rise or a downward fall along that line. The horizontal and the vertical motion cannot be actually differentiated in the large scale of the universe.

So if the laws of physics are valid in a gravitational field, as much as they are so on the earth surface, it implies that these are equally valid in an accelerated frame of reference. In this way Einstein generalized the new principle of relativity from the inertial frames of reference (STR) to the accelerated frames and thus completely broke with the Newtonian mechanics, valid only in the inertial systems.

Geometry of the Space-time Continuum

Very fine!

But what about the gravitational field? How is mechanical motion effected in such a field? If, contrary to Newton's explanation, the sun does not pull the earth with an attractive force, then what makes the earth move? What makes all planets move? If there is no attraction of the earth at a distance, why does a ball thrown upwards from the earth surface come back on it?

The proper scientific explanation of all this constitutes the most abstruse part of the GTR and, unlike STR, involves quite intricate mathematics. In this respect Einstein's picture is as much complicated as Newton's was mathematically simple and easily understandable. Einstein himself admitted: "The problem of formulating physical laws for every c. s. [co-ordinate system] was solved by the so-called *general relativity theory*; the previous theory, applying only to inertial systems, is called the *special relativity theory*. … But in sketching the way in which it was accomplished we must be even vaguer than we have been so far. New difficulties arising in the development of science force our theory to become more and more abstract. Unexpected adventures still await us. But our final aim is always a better understanding of reality. Links are added to the chain of logic connecting theory and observation. To clear the way leading from theory to experiment of unnecessary and artificial assumptions, to embrace an ever wider region of facts, we must make the chain longer and longer. The simpler and more fundamental our assumptions become, the more intricate is our mathematical tool of reasoning; the way from theory to observation becomes longer, more subtle, and more complicated."[7]

But we shall skip the tensor matrix mathematical representation of the new theory and try to present a rough sketch of the picture in simple metaphorical figures borrowing Einstein's own popular description.

This description owes its development to the concept of the geometry of space. There is an interesting thing about gravitation which attracted Einstein's attention. An electrical or magnetic field is temporary, transient and conditional, subject to the free play of an electric charge or a magnetic pole. If we screen the charge or the pole with a conducting medium or neutralize the charge or the polarity, the field will be destroyed. But a gravitational field cannot be so easily screened or destroyed, except in a very limited scale, in the case of free fall. What is the source of the permanence of gravitational effects? In quest of an answer to this question Einstein came to the conclusion that the gravitational property of matter is embedded in the geometric characteristics of its existence in space-time. Hence he turned his attention to the geometry of space.

As long as man's knowledge was confined to Euclidean geometry, he believed in the ideal atomization of space in terms of ideal geometric units like point, straight line, plane surface, square, cube, etc., abstracted from real material bodies. Geometry was considered distinct and distinguishable from both matter and space. Any part of the space could be severed from its totality and fitted with the geometrical figures separately developed and studied.

Ever since the second quarter of the nineteenth century, Gauss, Lobachevsky, Riemann, and others became conscious about the limitations of the Euclidean geometry. They embarked on the look out for a geometry that really fits with the real properties of the space. This led to the emergence of the non-Euclidean geometries of the space.

Let us cite a suitable example to clarify the point.

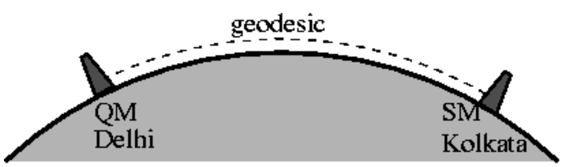

Fig. 4

The school geometry book teaches us that the shortest distance between two points is a segment of the straight line drawn through them. And when we measure a distance of, say, two meters or even fifty meters, we find no fault in it. Now suppose, we want to measure the shortest distance between Kolkata and Delhi (about 1400 kilometers). Can it be represented as a straight line? If we take a rigid and straight rod sufficiently long to cover the distance (say 1500 km) and place one end to the foot of the Shaheed Minar in Kolkata, the other end will not touch the foot of the Qutub Minar in Delhi, but its apex (as shown in fig. 4). In order to touch the foot of the QM we have to take a flexible nylon tape, which will spread along the actual spatial line over the surface of the earth (which is called the *geodesic*) within Indian territory and thereby describe the shortest distance as a curved line.

Thus there is no straight line in real space. What we see and draw as a straight line is only a rough approximation within very limited range, as an idealized construction by us.

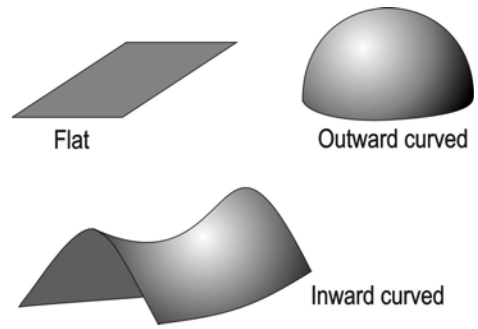

Fig. 5

Note that this spatial measurement is so because of the geometry of the earth's surface and this geometry is on its turn determined by the physical characteristics of the earth. In this way space and geometry are found to be fused together on the basis of the material bodies they represent. Already in the late nineteenth century, the British mathematician, William K. Clifford, after a thorough survey of the non-Euclidean geometries, had put forward an interesting idea that the physical properties of matter and the curvature of the geometry of the space might be related to each other. What appear as behaviours of material things and physical actions may well be due to the changing curvature of the space in time. For, there is no space apart from and without matter. And every geometry, if it is true, must represent the characteristics of some or other real space.[8] Since there are different varieties of space in the universe (flat, curved inward or outward), there are many variants of geometry discovered by man so far (see fig. 5).

Einstein had been deeply intrigued by this intuitive idea. But there were still many miles to go. The space Clifford and others were talking about is a three-dimensional one related to the positional description of matter. But already from the STR we know that matter is ever in motion and time is inseparable from the moving matter, and therefore from space. The correct geometrical description of matter, therefore, involves space and time together, represented as the four-dimensional space-time continuum. Immediately after the publication of the STR paper, Hermann Minkowsky developed the mathematical formalism of this 4-D space-time manifold. However, this manifold was an extension of a flat surface in the Euclidean geometry at a higher order of dimensions. Every point on this surface represented uniform motion and therefore an ideal inertial 4-D co-ordinate system. On the contrary, the points on the space-time manifold of the universe are points on the gravitational fields of matter, linked with accelerated systems; so they cannot be identical, for every point represents a unique motion. Hence the kinematics of bodies in the GTR requires a special kind of geometry with a changing curvature of its four-dimensional manifold.

Again we face a problem here: how to translate the abstract picture into a concrete form. We can draw only the two-dimensional space on paper. We can visualize three-dimensional space in the things we see all around us. But it is quite difficult to visualize the picture of a four-dimensional space in any concrete form. It can be presented only in an abstract mathematical formalism. Just as we can give only a rough idea of a three-dimensional figure on a two-dimensional surface like a plane paper, similarly let us try to depict the physical behaviour of a body in this four dimensional space-time structure with a simile of a three-dimensional space.

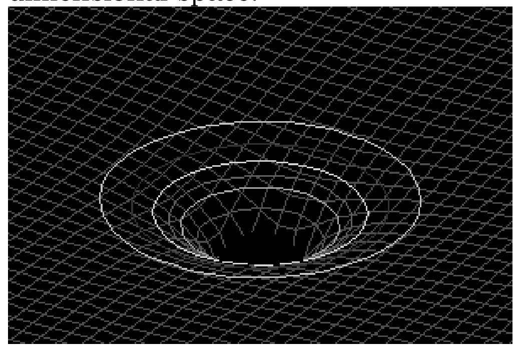

Fig. 6

Consider a vast sheet of canvas spread and suspended over a large area. Place a heavy wooden ball in it. It will be distorted at the pressure of the ball, in the middle where the ball will ultimately repose. Bring a piece of small marble in any peripheral side of the sheet of canvas. It will slowly and gradually slide along the distorted curvature of the sheet towards the ball (fig. 6). Common sense tells us that the speed of the marble will go on increasing the nearer it slides towards the ball.

Einstein pointed out, a similar kind of thing (at a higher order) happens in the case of the four-dimensional space-time structure. The flatness of the space-time continuum is distorted by the presence of a material body, which gives birth to a curvature of the four-dimensional surface. Another body coming nearby in this curvature will move along the inclination of the surface towards this body. The nearer it approaches the former body, the speedier its movement becomes. We who are untutored about the geometry of this four-dimensional space-time continuum will describe the same phenomenon as the fall of the latter to the former owing to its gravitational pull.

Since there are innumerable objects in this 4-D space of the universe, it is distorted throughout in such a way that different bodies will move in it along the shortest distance over the surface without falling upon one another. Not only ordinary material objects like the planets, satellites or the comets, but even light rays (consisting of photons) from a luminous object will propagate along such a line over the curvature of the 4-D surface near another massive body. It is on this basis that Einstein had predicted the bending of light rays from a distant star while coming by the sun (see fig. 7). This phenomenon cannot be usually seen on account of the brightness of the sun. It can be observed only on a total solar eclipse (TSE) day. And Einstein had calculated the angle of this bending as 1.75 arc-second on the basis of his mathematical treatment of the subject. An opportunity to test it presented itself very soon, when a TSE occurred on 29 May 1919. A joint committee of scientists formed at the initiative of Eddington by the Royal Society and the Royal Astronomical Society of England, sent two expedition teams – one to the Principe Island in the Gulf of Guinea, near West Africa, another to Sobral in Northern Brazil. They took rigorous photographs of the Constellation of Hyades, which at the time fell near the earth-sun line of sight. After a few months they again photographed the star when it had change position in the sky. Comparison of the two sets of photographs revealed a deviation in the range 1.43-2.7 arc-seconds, which was quite in good agreement with the predicted value. It may be noted here that a similar deviation was predicted on the basis of Newton's theory of gravitation, first by a German astronomer, Soldner in 1800, and later by others, yielding a result around 0.8 arc-second, which far from matched the observed mean value. It was no doubt a great triumph of the GTR over the previous theory.[9] Its another remarkable success was to produce the exact value of the precession of the perihelion of Mercury's orbit over time, which Newton's theory could not correctly account for.

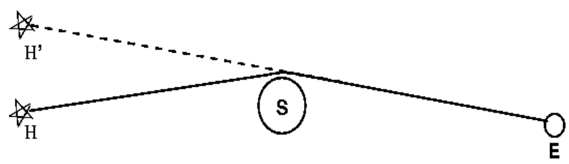

Fig. 7

Rumour has it that when in 1919 Arthur Eddington had been passing a sleepless night to observe the bending of light near the sun on the day of a solar eclipse and check the prediction, Einstein slept comfortably well in his house fully confident of the result.

In the GTR Einstein has finally combined the four-dimensional space-time continuum with matter of the universe, proved gravitation as a geometric property of this four dimensional manifold, and solved a number of long standing mysteries of nature.

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